RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2639

Mechanics 3

Tuesday **25 JANUARY 2005** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

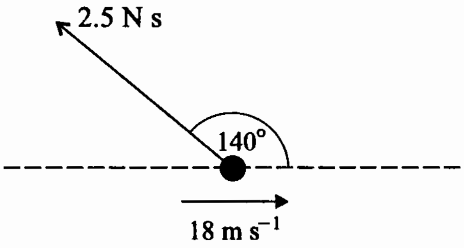
- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\,\text{m}\,\text{s}^{-2}$.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1



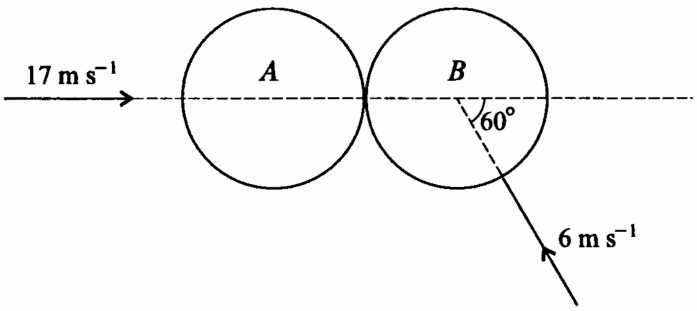
A hockey ball of mass 0.2 kg is moving with speed 18 m s^{-1} when it is hit by a stick. The ball receives an impulse of 2.5 N s at an angle of 140° to its initial direction of motion (see diagram). Find the speed of the ball immediately after it has been hit. [4]

2 A particle of mass m is moving in a complete vertical circle of radius a on the smooth inside surface of a fixed hollow sphere of internal radius a. Air resistance may be neglected. Show that

(i) when the particle is at the highest point of the circle, its speed is at least $\sqrt{(ag)}$, [2]

(ii) when the particle is at the lowest point of the circle, the normal reaction acting on the particle is at least $6mg$. [5]

3



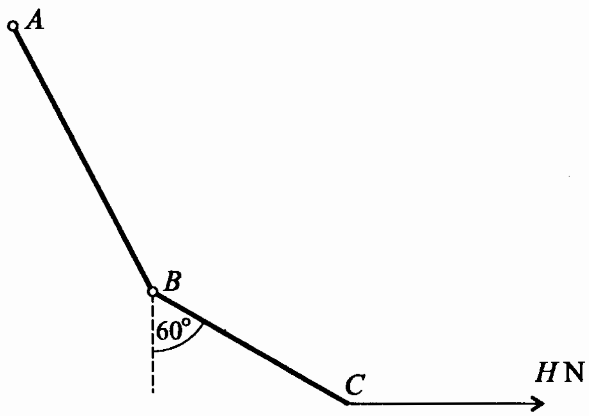
Two smooth spheres A and B, of equal masses and equal radii, are moving on a horizontal surface when they collide. Immediately before the collision, A has velocity 17 m s^{-1} along the line of centres, and B has velocity 6 m s^{-1} at an angle of 60° to the line of centres (see diagram). The coefficient of restitution between the spheres is 0.6. Find the speed of each sphere immediately after the collision. [8]

4 Two fixed points P and Q are 0.9 m apart on a smooth horizontal table. A particle X of mass m kg is connected to P by a spring of natural length 0.4 m and modulus of elasticity 60 N. The particle X is also connected to Q by a spring of natural length 0.5 m and modulus of elasticity 45 N. The particle X is moving along part of the line PQ, and air resistance may be neglected.

(i) Show that the motion of X is simple harmonic. [5]

(ii) Given that X is oscillating with period 0.48 s, find m. [3]

5



Two uniform rods AB and BC are freely jointed to each other at B, and AB is freely jointed to a fixed point at A. A horizontal force H newtons is applied at C and the rods are in equilibrium in a vertical plane with BC making an angle of $60°$ with the downward vertical (see diagram). The rod AB has length 1.6 m and weight 24 N; the rod BC has length 1.2 m and weight 18 N.

(i) Find H. [3]

(ii) Find the horizontal and vertical components of the force acting on AB at B. [2]

(iii) Find the angle which AB makes with the vertical. [4]

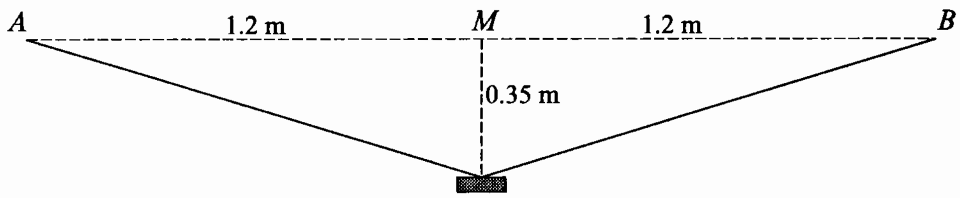
6 A stone of mass 0.1 kg is thrown vertically downwards with initial speed 6 m s^{-1} from a bridge over a river. After t seconds the speed of the stone is $v \text{ m s}^{-1}$. While the stone is falling the only forces acting on it are its weight and air resistance of magnitude $0.02v$ newtons.

(i) Show by integration that $v = 49 - 43e^{-0.2t}$. [7]

(ii) Given that the stone reaches the river 2.5 s after being thrown, find the height of the bridge above the river. [4]

[Question 7 is printed overleaf.]

7



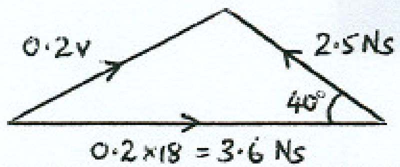
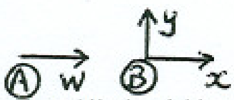
A brick of mass m kg is attached to two elastic strings, each having natural length 0.8 m and modulus of elasticity 112 N. The other ends of the strings are attached to fixed points A and B which are 2.4 m apart on the same horizontal level. The brick hangs in equilibrium 0.35 m vertically below M, the mid-point of AB (see diagram).

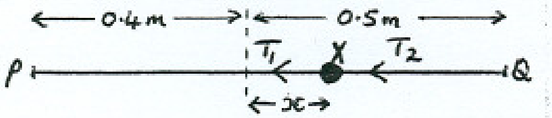
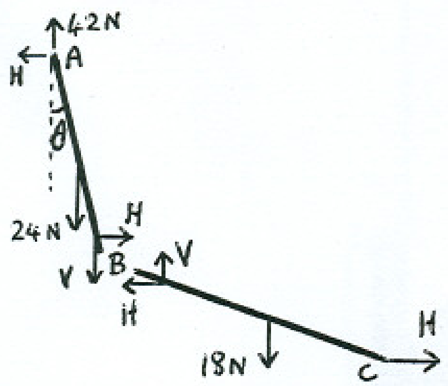
(i) Show that $m = 3.6$. [4]

While in this equilibrium position, the brick is given an impulse so that it begins to move with speed 3 m s^{-1} vertically upwards.

(ii) Find the speed of the brick when it passes through M. [6]

(iii) State three modelling assumptions you have made when answering this question. [3]

1	$\begin{pmatrix}2.5\cos 140\\2.5\sin 140\end{pmatrix} = 0.2\begin{pmatrix}v_1\\v_2\end{pmatrix} - 0.2\begin{pmatrix}18\\0\end{pmatrix}$	M1 A1	Equation involving impulse and momentum (one component sufficient) Both component equations correct
	$v_1 = 8.424,\quad v_2 = 8.035$		
	Speed $= \sqrt{v_1^2 + v_2^2}$	M1	
	$= 11.6\ \text{m s}^{-1}$	A1 4	
	OR 	M1 A1	Impulse / momentum triangle Correct triangle
	$(0.2v)^2 = 3.6^2 + 2.5^2 - 2\times 3.6\times 2.5\cos 40$	M1	
	$v = 11.6$	A1	
2 (i)	At highest point, $R + mg = m\dfrac{u^2}{a}$	M1	Or $mg = m\dfrac{u^2}{a}$
	$u^2 = ag + \dfrac{Ra}{m} \geq ag$, so $u \geq \sqrt{ag}$	A1 (ag) 2	Or $u = \sqrt{ag}$
(ii)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mg(2a)$	M1 A1	Using conservation of energy from highest point to lowest point
	When $u^2 = ag$, $v^2 = 5ag$	A1	
	$R - mg = m\dfrac{v^2}{a}$	M1	
	$R = 6mg$	A1 (ag) 5	
3	Velocities after collision 		
	$y = 6\sin 60\quad (= 3\sqrt{3} \approx 5.196)$	B1	
	$mw + mx = m(17) - m(6\cos 60)$ $(\ w + x = 14\ )$	M1 A1	Conservation of momentum
	$x - w = 0.6(17 + 6\cos 60)$ $(\ x - w = 12\ )$	M1 A1	Restitution equation
	$w = 1,\quad x = 13$	M1	Obtaining w or x
	Speed of A is $w = 1\ \text{m s}^{-1}$	A1	
	Speed of B is $\sqrt{x^2 + y^2} = 14\ \text{m s}^{-1}$	A1 8	

4 (i)			
	When displaced distance x from equilibrium,		
	Tension in PX is $T_1 = \frac{60}{0.4}x \quad (=150x)$	M1 A1	Using $\frac{\lambda x}{l}$
	Compression in XQ is $T_2 = \frac{45}{0.5}x \quad (=90x)$	A1	
	$-T_1 - T_2 = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$	M1	
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{240}{m}x$, hence motion is SHM	A1 **5**	
(ii)	Period is $2\pi\sqrt{\frac{m}{240}}$	B1 ft	ft from eqn of form $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2x + k$
	$2\pi\sqrt{\frac{m}{240}} = 0.48 \Rightarrow m = 1.40$	M1A1 **3**	
	OR $\omega = \frac{2\pi}{0.48}$ B1		
	$\frac{2\pi}{0.48} = \sqrt{\frac{240}{m}} \Rightarrow m = 1.40$ M1A1		
5 (i)			
	Moments about B for BC	M1	Moments equation for BC
	$H(1.2\cos 60) - 18(0.6\sin 60) = 0$	A1	Correct moments equation
	$H = 15.6$	A1 **3**	
(ii)	Horizontal component $= H = 15.6$ N (to right)	B1 ft	Directions must be clear
	Vertical component $= V = 18$ N (downwards)	B1 **2**	If B0, give B1 for 15.6 (ft) and 18
(iii)	Moments about A for AB	M1	Moments equation for AB (or ABC)
	$H(1.6\cos\theta) - V(1.6\sin\theta) - 24(0.8\sin\theta) = 0$	A2	Give A1 if one error
	$\theta = 27.5°$	A1 **4**	

6 (i)	$0.1\times 9.8 - 0.02v = 0.1\frac{dv}{dt}$ $t = \int \frac{1}{9.8-0.2v}dv$ $= -5\ln(9.8-0.2v) + C$ $v = 6$ when $t = 0 \Rightarrow C = 5\ln 8.6$ $t = -5\ln(9.8-0.2v) + 5\ln 8.6$ $e^{0.2t} = \frac{8.6}{9.8-0.2v}$ $9.8 - 0.2v = 8.6e^{-0.2t}$ $v = 49 - 43e^{-0.2t}$	M1 A1 M1 A1 M1 M1 A1 (ag) 7	Using N2L to obtain a diff eqn separation of variables $+C$ not required Must be correctly obtained
(ii)	$s = \int_0^{2.5} v\,dt = \left[49t + 215e^{-0.2t} \right]_0^{2.5}$ $= (122.5 + 130.4) - (215)$ $= 37.9$ m	M1 A1 M1 A1 4	Integrating v For $49t + 215e^{-0.2t}$ Using both limits (or finding constant of integration, then putting $t = 2.5$) Accept 38 m
7 (i)	Length of each string is $\sqrt{1.2^2 + 0.35^2} = 1.25$ Tension $T = \frac{112\times 0.45}{0.8}$ $(= 63)$ $2T\cos\theta = mg$ where $\cos\theta = \frac{0.35}{1.25}$ $m = \frac{2\times 63\times 0.28}{9.8} = 3.6$	B1 M1 M1 A1 (ag) 4	 Must be correctly obtained
(ii)	Initial EE is $2\times\frac{112\times 0.45^2}{2\times 0.8}$ $(= 28.35)$ Final EE is $2\times\frac{112\times 0.4^2}{2\times 0.8}$ $(= 22.4)$ By conservation of energy $\frac{1}{2}\times 3.6(3^2 - v^2) + (28.35 - 22.4) = 3.6\times 9.8\times 0.35$ $v = 2.33$ ms^{-1}	M1 A1 A1 M1 A1 A1 6	Using $\frac{\lambda x^2}{2l}$ (Award A1A1 for elastic energies if only one string is considered) Equation involving EE, KE and PE
(iii)	e.g. Brick is a particle No air resistance Strings are light Strings obey Hooke's law *or* No energy is lost in the strings	B1B1B1 3	For three assumptions